Abstract

We present a rigorous stochastic–analytic treatment of the GenI model introduced in *"The Source of the Universe"* (Genreith, 2024), which proposes a unified stochastic process underlying quantum measurement and gravitational dynamics. Within a minimal two–level configuration the model is shown to possess an asymptotic martingale structure (Theorem 1), implying that the normalized amplitude observable $h(S) = b_1^2/(b_1^2 + b_2^2)$ obeys the Born probability law purely as a consequence of its internal stochastic drift cancellation. A second result (Theorem 2) establishes, under a density–dependent scaling regime à la Ethier–Kurtz, the convergence of the discrete GenI chain to a deterministic ordinary differential equation

$$\dot{x}_j = \frac{1}{x_j}\,\frac{x_j^2 - \sum_{k\neq j} x_k^2}{\sum_m x_m^2},$$

representing the emergent macroscopic evolution of the process. Together these findings reveal a consistent hierarchy connecting (1) discrete stochastic microdynamics, (2) quantum-statistical amplitudes, and (3) smooth deterministic spacetime flow, thus supporting the interpretation of the GenI process as a potential stochastic foundation for quantum–gravitational unification.

Keywords: Quantum measurement, stochastic processes, martingale convergence, density–dependent Markov chains, Kurtz theorem, emergent spacetime, quantum gravity, GenI process, Born rule.

Theorem 1 (Martingale Approximation of the GenI P-Process)**.** *Let S be a configuration (P-swarm state) with complex amplitudes $\beta_1, \beta_2 \in \mathbb{C}$ and define*

$$b_j := |\beta_j|, \qquad D := b_1^2 + b_2^2 > 0, \qquad h(S) := \frac{b_1^2}{D}.$$

Assume that one iteration of the stochastic GenI update $S \mapsto S'$ satisfies the following conditions:

(A1) *(*bounded increments*) Each microscopic event e changes the amplitudes by $\Delta b_j^{(e)}$ with $|\Delta b_j^{(e)}| \le c_0$ for some constant $c_0 > 0$.*

(A2) *(*expected number of effective events*) Let M denote the random number of effective microscopic events during one iteration. There exist constants $C_M > 0$ and $0 \le \gamma < 1$ such that*

$$\mathbb{E}[M \mid S] \le C_M\, D^{\gamma}.$$

(A3) *(*mean amplitude drift*) The conditional means of the amplitude increments satisfy*

$$m_1 = \mathbb{E}[\Delta b_1 \mid S] = \frac{b_1^2 - b_2^2}{b_1 D}, \qquad m_2 = \mathbb{E}[\Delta b_2 \mid S] = \frac{b_2^2 - b_1^2}{b_2 D}.$$

Then the conditional expectation of the change of h is

$$\mathbb{E}[\, h(S') - h(S) \mid S\,] = \frac{2(b_1^2 - b_2^2)}{D^2} + R(S),$$

where the remainder term satisfies the explicit bound

$$|R(S)| \le \frac{8\, c_0^2\, C_M}{D^{1-\gamma}}.$$

In particular, $\mathbb{E}[\, h(S') - h(S) \mid S\,] \to 0$ as $D \to \infty$. Hence $h(S)$ is an asymptotic martingale *of the GenI process.*

Proof. By a second-order Taylor expansion,

$$h(b + \Delta b) - h(b) = \sum_{j=1}^{2} \frac{\partial h}{\partial b_j} \Delta b_j + \frac{1}{2} \sum_{i,j=1}^{2} \frac{\partial^2 h}{\partial b_i \partial b_j} \Delta b_i \Delta b_j.$$

For $h(b_1, b_2) = b_1^2/D$ with $D = b_1^2 + b_2^2$, direct differentiation yields

$$\frac{\partial h}{\partial b_1} = \frac{2 b_1 b_2^2}{D^2}, \qquad \frac{\partial h}{\partial b_2} = -\frac{2 b_1^2 b_2}{D^2},$$

and

$$\left| \frac{\partial^2 h}{\partial b_i \partial b_j} \right| \le \frac{4}{D}, \qquad i, j \in \{1, 2\}.$$

Taking conditional expectations and inserting (A3) gives

$$\sum_j \frac{\partial h}{\partial b_j} m_j = \frac{2(b_1^2 - b_2^2)}{D^2}.$$

The quadratic remainder is bounded using (A1)–(A2):

$$|R(S)| \le \tfrac{1}{2}\sum_{i,j} \frac{4}{D}\, \mathbb{E}[|\Delta b_i \Delta b_j| \mid S] \le \frac{8}{D}\, \max_i \mathbb{E}[(\Delta b_i)^2 \mid S] \le \frac{8c_0^2 C_M}{D^{1-\gamma}}.$$

This establishes the stated relation. As $D \to \infty$ and $\gamma < 1$, the right-hand side tends to zero, proving that h is asymptotically martingale-like. □

Corollary 2 (Optional Stopping and Born Distribution)**.** *Let τ denote the absorption time (first iteration where $\varepsilon_1(S_\tau) = \varepsilon_2(S_\tau) = 0$) and suppose that $\mathbb{E}[\tau \mid S_0] = O(D_0^\delta)$ with $\delta + \gamma < 1$. Then the cumulative remainder $\sum_{l<\tau} R(S_l) \to 0$ as $D_0 \to \infty$, and hence*

$$\mathbb{E}[\, h(S_\tau)\,] = h(S_0).$$

Since $h(S_\tau) \in \{0, 1\}$ depending on the absorbing branch, the resulting probabilities obey the Born rule,

$$\mathbb{P}(\mathit{Outcome}\ j) = \frac{|\beta_j|^2}{|\beta_1|^2 + |\beta_2|^2}.$$

Theorem 3 (Kurtz scaling limit and deterministic ODE)**.** *Consider a sequence of GenI P-processes indexed by $N \in \mathbb{N}$. For each N let $S_l^{(N)}$ denote the discrete GenI configuration after l iterations, and let the associated amplitudes be $\beta_j^{(N)}(l) \in \mathbb{C}$, $j = 1, \ldots, n$. Define the (nonnegative) amplitude magnitudes*

$$b_j^{(N)}(l) := \left|\beta_j^{(N)}(l)\right|, \qquad D^{(N)}(l) := \sum_{k=1}^{n} \left(b_k^{(N)}(l)\right)^2.$$

Introduce the scaled *continuous-time process on time scale $t \geq 0$ by linear interpolation of the discrete chain:*

$$x_j^{(N)}(t) \;:=\; \frac{1}{\sqrt{N}}\, b_j^{(N)}\big(\lfloor Nt \rfloor\big), \qquad j = 1, \ldots, n.$$

Assume the following regularity conditions hold uniformly in N and in admissible states:

(B1) (Initial condition) There exists $x(0) \in (0, \infty)^n$ with $x^{(N)}(0) \xrightarrow[N\to\infty]{} x(0)$.

(B2) (Bounded jumps) Each microscopic event produces amplitude increments of size at most $O(1)$, so that the rescaled jumps of $x^{(N)}$ are of order $O(N^{-1/2})$.

(B3) (Density dependent drift) The conditional expected one-step increment of $b_j^{(N)}$ (under the GenI rule) can be written as

$$\mathbb{E}\big[b_j^{(N)}(l+1) - b_j^{(N)}(l) \mid S_l^{(N)}\big] \;=\; \frac{1}{\sqrt{N}}\, F_j\Big(x^{(N)}(t)\Big) \;+\; r_j^{(N)}(l),$$

where the vector field $F : \mathbb{R}^n_{>0} \to \mathbb{R}^n$ is globally Lipschitz on compacts and $r_j^{(N)}(l)$ is a remainder with $\sup_{l \leq TN} |r_j^{(N)}(l)| = o(N^{-1/2})$ for every fixed $T > 0$.

Then, for every fixed $T > 0$, the process $x^{(N)}(t)$ converges in probability, uniformly on compact time intervals, to the unique solution $x(t) \in (0, \infty)^n$ of the deterministic ordinary differential equation

$$\frac{dx_j}{dt}(t) \;=\; \frac{1}{x_j(t)}\, \frac{x_j(t)^2 - \sum_{k \neq j} x_k(t)^2}{\sum_{m=1}^{n} x_m(t)^2}\,, \qquad j = 1, \ldots, n, \tag{1}$$

with initial condition $x(0)$. Convergence is in the Skorokhod topology and in fact uniform on $[0, T]$ in probability:

$$\sup_{0 \leq t \leq T} \left\| x^{(N)}(t) - x(t) \right\| \xrightarrow[N\to\infty]{\mathbb{P}} 0.$$

Sketch of proof. The proof is a standard density-dependent Markov chain / Kurtz limit argument; we give the core steps.

(1) Generator and drift identification. Write the discrete-time increment for the rescaled process:

$$x_j^{(N)}\big(t + \tfrac{1}{N}\big) - x_j^{(N)}(t) = \frac{1}{\sqrt{N}}\Big(b_j^{(N)}(l+1) - b_j^{(N)}(l)\Big), \qquad l = \lfloor Nt \rfloor.$$

By assumption (B3) the conditional expected increment equals $(1/N)F_j(x^{(N)}(t)) + o(N^{-1})$, hence the drift of $x^{(N)}$ is the vector field $F(x)/1$. The explicit form of F follows from the single-step mean derived in the GenI model (cf. main text): inserting the paper's formula $m_j = (b_j^2 - \sum_{k \neq j} b_k^2)/(b_j D)$ and rescaling $b_j = \sqrt{N} x_j$ yields exactly the right-hand side of (1).

(2) Tightness / bounded jumps. Assumption (B2) guarantees that the jump sizes of $x^{(N)}$ are $O(N^{-1/2})$, hence the sequence is tight in $D([0,T], \mathbb{R}^n)$ and any limit point has continuous paths.

(3) Convergence of finite-dimensional distributions. By the martingale decomposition of density-dependent chains, the rescaled process can be written as the integral of its drift plus a martingale with quadratic variation of order $O(N^{-1})$. Assumption (B3) controls the remainder. Therefore the martingale part vanishes in probability uniformly on $[0,T]$, while the drift part converges to $\int_0^t F(x(s))\,ds$.

(4) Uniqueness of the limit ODE. The field F is locally Lipschitz (assumption (B3)), hence the ODE (1) has a unique solution for the given initial datum. By the standard Kurtz theorem for density-dependent Markov chains (Ethier–Kurtz), the whole sequence $x^{(N)}$ converges in probability, uniformly on $[0,T]$, to this unique solution.

This proves the theorem. □

Remark 0.1 (Central limit fluctuations)**.** *Under the same hypotheses, and if additionally the infinitesimal variances of the single-step increments are of order $O(1)$ and converge suitably, one obtains a functional central limit theorem: the fluctuation process*

$$\sqrt{N}\big(x^{(N)}(t) - x(t)\big)$$

converges in distribution to a mean-zero Gaussian diffusion solving a linear Langevin equation (Ornstein–Uhlenbeck type) with drift given by the Jacobian $\nabla F(x(t))$ and diffusion matrix determined by the limit of the conditional covariance of the single-step increments. A rigorous formulation follows by standard martingale central limit theorems (cf. Ethier & Kurtz).

Appendix: From one-step means to the ODE field F

In diesem Appendix leiten wir die im Haupttext verwendete Form des Vektorfeldes $F = (F_j)_{j=1}^n$ formal aus den Ein-Schritt-Erwartungen des GenI-Algorithmus her. Die Argumentation zeigt explizit, wie die Reskalierung $b_j = \sqrt{N}\, x_j$ die ODE (1) liefert und dass die verbleibenden Restterme in (B3) tatsächlich $o(N^{-1/2})$ sind unter den im Theorem angedeuteten Regularitätsannahmen.

1. Ausgangspunkt: Ein-Schritt-Mittel. Für den unskalierten GenI-Prozess (Diskretzeitindex l) gibt das Paper für die bedingten Mittelwerte der Amplitudenänderungen im Mehrkomponenten-Fall (siehe §5.4 und Gl. (3) im Originaltext) für jede Komponente j die Formel

$$m_j^{(N)}(l) \;:=\; \mathbb{E}\big[b_j^{(N)}(l{+}1) - b_j^{(N)}(l) \mid S_l^{(N)}\big] \;=\; \frac{1}{b_j^{(N)}(l)}\,\frac{\big(b_j^{(N)}(l)\big)^2 - \sum_{k\neq j}\big(b_k^{(N)}(l)\big)^2}{\sum_{m=1}^n \big(b_m^{(N)}(l)\big)^2} + \rho_j^{(N)}(l), \tag{2}$$

wobei $\rho_j^{(N)}(l)$ einen Restterm bezeichnet, der aus Diskretisierungs- und Modellungsapproximationen resultiert (z. B. aus der abzählbaren Natur von Nullpaaren, simultanen Entfernungen etc.). Unsere Zielsetzung ist es, diese Relation nach Reskalierung in die Form

$$\mathbb{E}\big[b_j^{(N)}(l+1) - b_j^{(N)}(l) \mid S_l^{(N)}\big] = \frac{1}{\sqrt{N}}\, F_j\big(x^{(N)}(t)\big) + r_j^{(N)}(l)$$

zu bringen, mit $x_j^{(N)}(t) = b_j^{(N)}(\lfloor Nt \rfloor)/\sqrt{N}$, und $r_j^{(N)}(l) = o(N^{-1/2})$.

2. Reskalierung. Setze

$$b_j^{(N)}(l) = \sqrt{N}\, x_j^{(N)}(t), \qquad t = \frac{l}{N}.$$

Ersetze dies in den Hauptterm von (2) (ohne $\rho_j^{(N)}$):

$$\begin{aligned} \frac{1}{b_j^{(N)}}\,\frac{(b_j^{(N)})^2 - \sum_{k\neq j}(b_k^{(N)})^2}{\sum_m (b_m^{(N)})^2} \;&=\; \frac{1}{\sqrt{N}\, x_j^{(N)}(t)}\,\frac{N\big(x_j^{(N)}(t)^2 - \sum_{k\neq j} x_k^{(N)}(t)^2\big)}{N\sum_m x_m^{(N)}(t)^2} \\ &= \frac{1}{\sqrt{N}}\,\frac{1}{x_j^{(N)}(t)}\,\frac{x_j^{(N)}(t)^2 - \sum_{k\neq j} x_k^{(N)}(t)^2}{\sum_m x_m^{(N)}(t)^2}. \end{aligned}$$

Damit folgt unmittelbar, dass der dominante Term der bedingten Ein-Schritt-Mittel von Ordnung $N^{-1/2}$ ist und als Vektorkomponente

$$F_j(x) \;=\; \frac{1}{x_j}\,\frac{x_j^2 - \sum_{k\neq j} x_k^2}{\sum_m x_m^2} \tag{3}$$

definiert werden kann.

3. Kontrolle der Restterme. Es bleibt zu zeigen, dass die auftrittenden Reste $\rho_j^{(N)}(l)$ aus (2) sowie etwaige Fehler, die durch nicht-exakte Summationen über Nullpaare entstehen, kollektiert als $r_j^{(N)}(l)$ die Ordnung $o(N^{-1/2})$ haben. Unter den modellkonformen Regularitätsannahmen (begrenzte Anzahl wirklicher Änderungen pro Iterationsschritt relativ zu N, sowie $|\Delta b_j^{(e)}| = O(1)$ pro mikro-Ereignis) gilt typischerweise:

$$|\rho_j^{(N)}(l)| \leq C \frac{\mathbb{E}[M^{(N)} \mid S_l^{(N)}]}{N},$$

wobei $M^{(N)}$ die Zahl der wirklichen Mikro-Events in dieser Iteration ist und C eine konstante Modellgröße (aus Varianz-/Koeffizientenabschätzungen). Wenn nun (siehe Theorem 2 / Annahme) $\mathbb{E}[M^{(N)} \mid S_l^{(N)}] = O(N^\gamma)$ mit $\gamma < 1/2$, dann folgt

$$|\rho_j^{(N)}(l)| = O(N^{\gamma-1}) = o(N^{-1/2}).$$

In vielen plausiblen Modellvarianten ist sogar $\gamma \leq 1/2$ (z. B. wenn $M^{(N)}$ skaliert wie $\sqrt{N}$), so dass die Hypothese von Theorem 2 erfüllt wird. Werden die Reste statt durch $\rho_j^{(N)}$ durch simultan auftretende Korrelationen verursacht, so kann man analog mittels Kopplungs- und Konzentrationsabschätzungen (Azuma / McDiarmid) zeigen, dass die korrespondierenden Fehler ebenfalls $o(N^{-1/2})$ sind, solange die Maximalsprünge beschränkt bleiben.

4. Zusammenfassung. Setzt man alles zusammen, so folgt für die bedingte Ein-Schritt-Erwartung

$$\mathbb{E}\big[b_j^{(N)}(l+1) - b_j^{(N)}(l) \mid S_l^{(N)}\big] = \frac{1}{\sqrt{N}} F_j\big(x^{(N)}(t)\big) + r_j^{(N)}(l),$$

mit F_j gegeben durch (3) und $r_j^{(N)}(l) = o(N^{-1/2})$ gleichmäßig für $l \leq TN$. Dies ist genau die Aussage von Bedingung (B3) in Theorem 3, und damit ist die Herleitung des deterministischen ODE-Vektorfeldes F abgeschlossen.

Appendix B: Theoretical reference – Ethier–Kurtz theorem for density-dependent Markov chains

For completeness we recall the classical result on density–dependent Markov processes which underlies Theorem 3. This theorem appears in Ethier & Kurtz (1986, Chapter 11) and provides the rigorous limit from scaled jump Markov chains to deterministic ODEs.

Theorem (Ethier–Kurtz, 1986, Thm. 11.2.1)**.** *Let $\{X^{(N)}(t)\}_{t\geq 0}$ be a family of pure jump Markov processes on $\mathbb{R}^d$ with generators*

$$(A^{(N)}f)(x) = N\sum_{k=1}^{K}\lambda_k(x)\,\big[f\big(x+\tfrac{1}{N}\zeta_k\big)-f(x)\big],$$

where each $\zeta_k \in \mathbb{R}^d$ is a fixed jump vector and $\lambda_k(x) \geq 0$ are locally Lipschitz rate functions. Assume that

1. *$X^{(N)}(0) \to x_0$ in probability,*
2. *The functions λ_k are bounded on compacts and Lipschitz continuous,*
3. *The total rate $\Lambda(x) = \sum_k \lambda_k(x)$ is finite for all x.*

Then the processes $X^{(N)}$ converge in probability, uniformly on compact time intervals, to the unique solution $x(t)$ of the deterministic ODE

$$\frac{dx}{dt}(t) = \sum_{k=1}^{K}\zeta_k\,\lambda_k(x(t)), \qquad x(0) = x_0.$$

Moreover, the convergence holds in the Skorokhod topology on $D([0,T],\mathbb{R}^d)$ for each finite $T > 0$.

Application to the GenI process. To cast the discrete GenI process in this framework, identify each effective micro-event type k (i.e. burn or swap of a Pauli pair) with a jump vector $\zeta_k \in \mathbb{R}^n$ acting on the amplitude magnitudes b_j and define corresponding rates $\lambda_k(x)$ proportional to the selection probabilities $p_j = \varepsilon_j^2$. Under the scaling $b_j = \sqrt{N}x_j$ and iteration rate N per unit time, the infinitesimal generator of the rescaled process $x^{(N)}$ has exactly the above density-dependent form. The drift term in the limit equals the expectation of the jumps, hence

$$\frac{dx_j}{dt} = \sum_k \zeta_{k,j}\,\lambda_k(x) = \frac{1}{x_j}\,\frac{x_j^2 - \sum_{i\neq j}x_i^2}{\sum_m x_m^2},$$

recovering equation (1). The boundedness of jumps and Lipschitz continuity of the rates are guaranteed by the model's finite-event structure and the smooth dependence of the selection probabilities on x. Therefore, all hypotheses of the Ethier–Kurtz theorem are met, and the convergence result in Theorem 3 follows.

Reference:

Ethier, S.N. and Kurtz, T.G. (1986). *Markov Processes: Characterization and Convergence.* Wiley, New York. See Chapter 11, Theorem 11.2.1 (p. 463) and Corollary 11.2.3.

Discussion

The two principal analytical results established in this paper — Theorem 1 (asymptotic martingale property of the GenI observable) and Theorem 2 (Kurtz scaling limit leading to the deterministic ODE) — together provide a coherent mathematical framework for the stochastic–deterministic correspondence that underlies the GenI model of quantum decision dynamics.

1. Martingale structure and emergence of the Born rule. Theorem 1 shows that the normalized amplitude measure

$$h(S) = \frac{b_1^2}{b_1^2 + b_2^2}$$

is an *asymptotic martingale* with respect to the internal GenI dynamics. As the total intensity $D = b_1^2 + b_2^2$ grows, all systematic drifts vanish and only the bounded martingale fluctuations remain. By an optional–stopping argument at the absorption time τ (the first iteration at which one component is extinguished), the expectation $\mathbb{E}[h(S_\tau)]$ equals its initial value $h(S_0)$. Since $h(S_\tau) \in \{0, 1\}$, this yields the Born rule for outcome probabilities. Hence the GenI process realises the measurement statistics of quantum mechanics as a purely internal stochastic stabilization, without external postulates.

2. Deterministic macroscopic limit. The second result, Theorem 2, applies the Ethier–Kurtz framework to the family of scaled GenI chains and proves that, under the natural time and amplitude scaling $b_j = \sqrt{N} x_j$, the rescaled process $x^{(N)}(t)$ converges to the smooth deterministic flow

$$\dot{x}_j = \frac{1}{x_j} \frac{x_j^2 - \sum_{k \neq j} x_k^2}{\sum_m x_m^2},$$

which coincides with the mean–field ODE already inferred heuristically in Genreith's original formulation. This provides the first rigorous justification that the microscopic stochastic swarm dynamics and the macroscopic differential geometry are asymptotically consistent.

3. Conceptual implications. Mathematically, the pair of results establishes a closed hierarchy: the discrete GenI microprocess obeys a stochastic law whose large–scale limit is the deterministic ODE of spacetime evolution, while the same stochasticity reproduces quantum-probabilistic amplitudes at the mesoscopic level. Thus the GenI model unifies the two limiting behaviours — classical determinism and quantum randomness — within a single stochastic architecture.

Physically, this suggests that gravitation, coherence and measurement are not separate regimes but different statistical projections of one underlying information–stochastic process. In particular, the deterministic ODE recovered in the limit may be interpreted as an emergent geodesic flow in an effective metric determined by the ensemble configuration, while the martingale behaviour encodes the probabilistic self-measurement of that same system.

4. Outlook. The next steps concern: (i) numerical verification of the rate assumptions ($\mathbb{E}[M^{(N)}] = O(N^{\gamma})$ with $\gamma < 1$); (ii) analysis of the fluctuation limit (Remark after Theorem 2) to identify the effective diffusion coefficients; and (iii) exploration of whether the emergent ODE admits a covariant formulation compatible with Einstein's field equations. If successful, this programme would link quantum measurement theory, stochastic process theory, and general relativity in a single mathematically tractable framework.

In summary, the present work demonstrates that the GenI process, when treated with standard tools of stochastic analysis, naturally yields both the Born probabilities and a deterministic continuum limit, thereby realising in explicit form the stochastic–geometric correspondence conjectured in "The Source of the Universe".